Size Relationships of Big History Objects: From the Universe to the Atomic Nucleus

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Abstract: Big History involves a variety of sizes from the atomic nucleus to the size of the universe. How can we make sense of this? A popular video (Eames 1977) about the power of 10 took a “picture” for every factor of 10 in distance with over 40 “stops” from a nucleus (not even the smallest distance we can talk about) to the edge of the universe. Instead, we will explore just nine steps in distances of things we know about: the atomic nucleus, the atom, a bacterium, a human, the Earth, the distance to the sun, the distance to the closest star, the size of the galaxy, and the size of the universe. Quite amazingly, many of these distances can be estimated with only a few fundamental physics constants. Also, the ratio of the distances of the larger to the next smaller is about the same (with some exceptions): about 150,000. How can this factor of 150,000 be visualized? It is about the ratio of a commercial jet’s height compared to your palm, or, using the American standard unit of a football field, the smaller distance would be about 1/32 of an inch.

Introduction

Besides the different time scales in Big History, quite a range also exists in sizes of objects involved in Big History. The time range includes the billions of years of the universe, stars, and planets; millions of years for species; thousands of years for historical processes; generations for humans and technology development; and seconds for DNA replication, thoughts, and computer calculations. The size scales cover the range of the universe as a whole to the atomic nucleus.

While one might think that there are so many physical phenomena involved in determining these sizes, any systematic discussion about this range would involve innumerous details and inputs from a range of fields. Instead, many in the late 1970s (e.g., Carr and Rees 1979; Press and Lightman 1983) explored these size relationships to uncover estimates that were based surprisingly on just a few constants and a few physical phenomena constraining the environments in which the objects would occur. Here those relationships are reviewed, and actual distances are compared: nine of the fundamental distances in Big History follow a pattern with a ratio of the sizes being about 150,000.

The path starts with the relationship between the atom and the size of the universe. Then intermediate sizes follow along with the smaller size of the proton.

Size of an Atom

The first distance to discuss is the size of an atom. While quantum mechanics may give details, the size can be estimated based on the need to balance the attraction between the charges of the electron and proton with the energy caused by the uncertainty principle. That is, as the negatively charged electron attempts to move closer to the positively charged proton, its velocity and momentum increase, but the product of uncertainty in distance and momentum remains constant (related to the Planck constant, P). This leads to a simple estimate of the size of an atom, which is denoted as ‘a’. With the mass of the electron being m, and the charge of the electron C, then balancing the energy leads to the estimate of $(PC)^2/m$, which gives about 10^{-10} meters (about a tenth of a nanometer). For reference, DNA is about one nanometer wide and the transistors in your computer or smartphone are about fifteen nanometers wide.

Many of the remaining distances can then be expressed as the product of this fundamental distance and some powers of two unitless parameters, G and A. The first (G) is just the ratio of the electromagnetic and gravitational forces between two protons (any distance). This has a numerical value of about $5x10^{-39}$. The second (A) is called the electromagnetic fine structure constant ($C^2/(Pc)$, where c is the speed of light), which is numerically equal to about 1/137, or 0.0073. This is related to the maximum charge of 137 protons in an atomic nucleus. (So far the highest charged nucleus has 118 protons and the highest natural nucleus is uranium with 92 protons.) The square of this parameter is the fraction of energy in the bound hydrogen atom compared to the mass-energy of the electron, mc^2. This energy can be estimated in a way similar to how the distance was derived in the paragraph above.

Size of the Universe

Now jump to consider the other end of the size spectrum, the size of the universe. To simplify matters, let B represent the ratio A/G, which is about $1.5x10^{36}$. Then, the size of the universe is about this many atomic distances ('a' from above), i.e., size of the universe (U) = Ba, which is about 15 billion light years, i.e., the distance light would travel from the beginning of the universe. However, since the universe has been expanding, the true size of the observable universe is about six times this distance. How is this simple formula derived, especially since the size of the universe changes with time (from a point in the Big Bang to the current size)? The derivation of this relationship requires an additional assumption that enough time had passed for the universe to generate enough carbon (and other elements) to make planets and life. Hence, this is really an estimate of the size of the universe when conditions are ready for life with many generations of stars.

Size of Galaxies

Making other assumptions (discussed below) about the conditions necessary for the formation of galaxies and planets leads to an estimated typical galaxy size of about A^2U. This shows that the size of galaxies relative to the universe is about the same fraction (0.005%) of the energy in the hydrogen atom compared to the electron mass-energy. This estimated galactic size is about four times larger than the size of the Milky Way (150,000 light-years), which is quite accurate, considering the simplicity of the estimate and the variability in galactic sizes.

The necessary assumptions consider the relative rates of cooling and gravitational condensation of a cloud of mostly hydrogen and helium gases that existed about a billion years after the Big Bang. As these clouds begin to condense because of gravity, this gravitational energy is converted to heat, which must be expelled from the clouds to maintain the condensation. If it is not expelled quickly enough, the pressure builds up and the gravitational force is canceled by the pressure. Much of the cooling is done through ionization of hydrogen, which is dependent on the energy of ionization that can be estimated from the size of the atom (as above). The largest mass at which this cooling mechanism works gives a galactic mass of about a trillion solar masses. Further discussion of this process and its relationships to other Big History eras can be found in a recent paper by Grinin and Grinin (2019).

Distance to Nearest Star

As these galaxies form from cloud collapse and merging of smaller galaxies, a net rotation of the stars around a common center can occur. The rotation tends to flatten out a galaxy just as a clump of pizza dough takes its shape as the baker throws it in the air with a good spin (similar to the way planets orbit around the sun in a disk). The mass of a typical star can be estimated based on the physics of nuclear reactions. This is done by balancing the thermal pressure from the nuclear fusion reactions in the star's core with the weight of the outer part of the star pushing in because of gravity. This estimation requires knowledge of very many fundamental aspects of physics including gravity, electromagnetism, both the strong and weak force, along with thermodynamics and quantum mechanics (as protons react through quantum "tunneling").

By knowing the mass of a typical star and galaxy, the number of stars can be estimated. Combining this with the disk size of the galaxy (as found above), an estimate for a typical distance between stars in a galaxy can be found. Note that in nature are many different types of galaxies and within each galaxy are different types of stellar environments such as the crowded central hub and the less dense spiral arms (like we are in) with fewer stars between the arms. This consideration of galactic size and number of stars can be used to find the average distance between stars, which is similar to the distance from the sun to its nearest star, Proxima Centauri, of 4.2 light years away.

Distance from Earth to Sun

The Earth is in the "Goldilocks" zone, better known as the habitable zone around the sun where the temperature is such that liquid water might form. While some supercomputers might be able to calculate some properties of water from first principles, it is unlikely that all the special and unique properties of this substance could easily be explained. The electrons around hydrogen and oxygen do their dance to create an unusual substance that floats when it freezes, can dissolve both salts and organic compounds, can easily evaporate, and can be moved around the world in clouds while occasionally condensing to water the planet.

The distance from the sun that this habitable zone incorporates depends mostly on the energy output from the sun. The paragraphs above mentioned that many aspects of physics were combined to estimate the mass of stars. With this mass, the rate of nuclear reactions in the core can be estimated and with that the energy output of typical stars. The energy per area falls off as the square of the distance from the sun. Some assumption needs to be made concerning how much energy would be absorbed by a planet while that remaining is reflected out into space. This absorption can be used to find the temperature by assuming that the planet comes into a balance between the energy being absorbed from the sun and the amount of energy being radiated back out into space at the planet's lower temperature. (This radiation is similar to the orange glow coming from a heated stove or toaster.) Knowing the mass and luminosity of a star leads to an estimate of a planet's temperature. This estimated distance is again close to the actual distance from the sun to Earth of 150 million kilometers.

Size of a Habitable Planet

Rocky planets are large enough that gravity tends to pull the rocky material into near spherical shapes. Gravity also helps hold atmospheres. The gases in the atmosphere might escape if gravity is too weak since the molecules move at fast speeds. The estimated rocky planet's radius that can hold an atmosphere at temperatures comfortable for life comes to about 4,000 km, which is about 60% of the Earth's radius but within 15% of the size of Mars.

To grasp an idea of the magnitude of the gravity needed, the average speed of a typical atmospheric molecule, N_2, can be easily found (about 500 m/s), whereas the speed of anything (rockets or molecules) to escape the Earth's gravity is about 11 km/s, that is, about twenty-two times the average velocity of molecules.

However, many other factors should be considered. For example, Mars currently has about 1% of the Earth's atmospheric pressure at its surface, and it is still losing it. On the other hand, Venus has a mass and a radius more similar to Earth but contains an atmosphere with a pressure nearly one hundred times larger. The atmospheres of Venus and Mars might have been more similar to the Earth long ago (Way 2019).

Size of a Human

Since we are looking for only an *estimate* for the size of a human, we will consider in general how big animals can grow on land. Many criteria, such as heat management, might be considered, but a significant aspect is that animals would not survive long if they were easily severely injured when they fell. Even at our size it is relatively common to break a bone or sprain a muscle, but a fall is usually not fatal until the height is about fifteen meters, i.e., about ten times a human's size. A key factor in this estimate is the strength of the material that might break compared to the planet's gravity. Some material properties can be estimated with slightly less simple models than considered earlier, while the gravity is based on a planet that can maintain its atmosphere (as above). Considering material of lengthy polymers gives an estimated size close to human-sized scale.

Size of a Cell

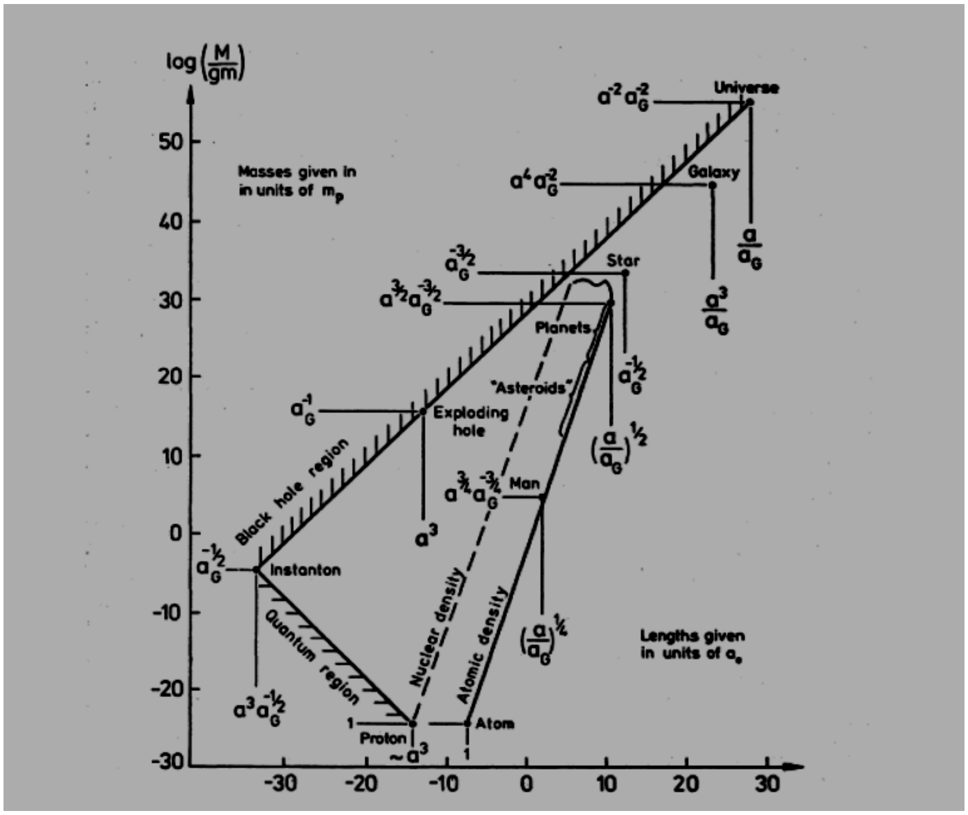
The basic unit of life is a cell. Bacteria are 0.2 to 10 microns but the common bacterium, *E.coli*, is a rod two microns long by one micron in diameter. Animal cells are between ten and a hundred microns. While these cells have membranes to separate the cellular interior from the outside, the smallest natural size of a water droplet (in fog, for example) is about ten microns. At this size the surface tension forces of the water molecules at the boundary are equal to the evaporative forces. This can be estimated with the intermolecular forces of hydrogen bonding (weaker than a covalent chemical bond) and the temperature of liquid water on Earth.

Size of an Atomic Nucleus

Finally, we consider the proton size (smallest atomic nucleus). A few approaches to estimating this size can be taken. One approach is to consider the range of the strong force keeping the three quarks in the proton together, but this strength is not included in the parameters considered so far. Another approach is to find the (Compton) wavelength of light, which has the same energy contained in a particle's mass. Then the proton size compared to the 'classical' electron size would be scaled by the ratio of electron and proton masses. However, this mass ratio (approximately 1/2000) is currently not explained. Despite no direct theory for this ratio, certain relationships between the electromagnetic and strong forces are needed for the existence of complex elements that are needed for life. This results in the conclusion that the proton size is much (about 100,000 times) smaller than the size of an atom.

Other properties (like mass) can also be estimated with these methods. The figure from Carr and Rees's (1979) paper shows the range of sizes (x-axis) and mass (y-axis) from the proton to the universe. Note that an additional item is labeled in the lower left of Figure 1. This is the "Instanton" at the

Figure 1. Estimates of relative sizes from mostly first principles (Source: Carr & Rees 1979)



Planck scale, which is the smallest size anything can be, considering gravity and quantum physics. It occurs at quite a qualitatively different location on the diagram, i.e., the vertex of the quantum and black hole regions. The size is about 10^{-35} m or twenty orders of magnitude smaller than the atomic nucleus.

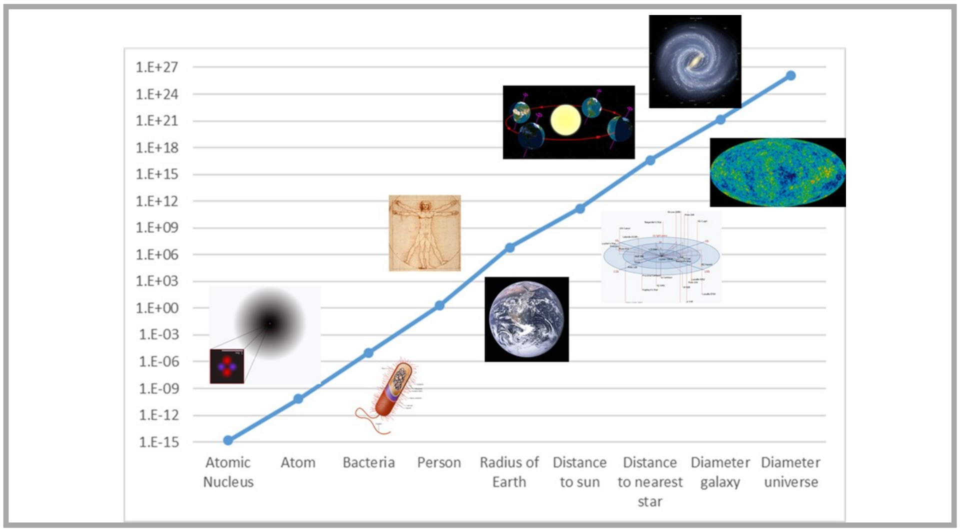
Comparison to Real Sizes

Now we can take a look at the real numbers and ratios of these distances and sizes (Table 1 and Figure 2). Not only can they be estimated with the few parameters used above, but the ratios from one item to the next in the sequence are somewhat constant, about 150,000. (One item in the list stands out as being a bit out of place. Using 10% of the radius of the Earth rather than the full radius is better.) This number makes sense because remember that we started with estimates of the sizes of an atom and the universe as being the ratio of Alpha to Alpha-G, which is about $1.5x10^{36}$. Since there are seven factors between these two, we can see that a factor of 150,000 applied seven times would lead to a factor of $1.7x10^{36}$. The Planck scale mentioned above would be another four of these scaling factors smaller than a nucleus.

Table 1. Actual Ratios of Distances and Sizes

Object	Size		Ratio	Factor Difference
Proton	1.0E-15	meter		
Atom	1.0E-10	meter	1.0E+05	-3%
Bacteria	1.0E-05	meter	1.0E+05	-3%
Person	1.5E+00	meter	1.5E+05	0%
Radius of Earth	6.0E+06	meter	4.0E+06	28%
Distance to Sun	1.5E+11	meter	2.5E+04	-15%
Distance to Nearest Star	4.2E+00	light year	2.6E+05	5%
Galactic Diameter	1.5E+05	light year	3.6E+04	-12%
Observable Universe Size	9.3E+10	light year	6.2E+05	12%
Factor difference = (log (Ratio) - log (R0)) / log (R0)				
That is, Ratio = R0 ^ (1+Factor Difference)				
R0=150,000				

Figure 2. Plotting data of Table 1 (Source of images: *Wikipedia*)



Another way of looking at this reveals some connections among the sizes. An item's size is the geometric average of the two items equally spaced above and below. For example, the size of a person should be about the geometric average of the size of an atom (two steps smaller) and the distance to the sun (two steps larger), i.e., the square root of 150 million kilometers x 0.1 nm. After making sure the units are the same, we end up with a size of about 4 m. This is about a factor of 2 too large, but it is quite amazing it is this close.

How, then, can we relate to a factor of 150,000? We definitely know the difference between $1 and $150,000, but are there some sizes to which we can relate? There seem to be. Commercial jets fly at about 6.5 miles, which is a bit over 10 km (equivalent to a million centimeters); 1/150,000 of this distance is about 7 cm (3 inches), a bit less than the width of a hand. The next time you are in an airplane at cruising altitude, look down and try to visualize someone's hand. (It is easier to see cars and then imagine the people in them). That ratio of sizes and distance, the size of a hand to the altitude is about the same as the ratio among these nine sizes and distances in Big History, i.e., nucleus to atom, atom to bacterium, bacterium to human, human to (smaller) Earth, Earth to sun, distance to sun compared to distance to the nearest star, distance of the nearest star to galactic size, and finally galactic size to the (observable) universe. Alternatively, using the American standard unit of a football field, the smaller distance would be about 1/32 of an inch.

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